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LETTER TO THE EDITOR

Statistics of the transmission coefficient and its dependence on a constant electric field in disordered one-dimensional conductors

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Abstract. Abrikosov's microscopic results for the transmission coefficient in disordered one-dimensional conductors are supplemented by an analysis of its distribution in the quasi-metallic regime and a discussion of the effect of a constant electric field. Our treatment is based on the method of invariant imbedding, which is shown to be equivalent to Abrikosov's method at zero field, for uniformly distributed phases.

The study of the statistical properties of the transmission coefficient t of disordered one-dimensional systems [1, 2] is of interest because, like other transport parameters, t reflects electron localisation effects. It is also of interest because of the uncertainty surrounding the definition of the conductance of a finite one-dimensional disordered conductor. Indeed, some authors [1, 3, 4] have recently identified the dimensionless conductance g with the transmission coefficient rather than with the familiar Landauer expression [1]

$$g = \rho^{-1} = t/(1 - t) \tag{1}$$

where ρ is the dimensionless resistance. While the two definitions of g differ significantly only in the large-conductance regime ($t \to 1$), they generally lead to quite different statistical properties, since, for example, the mean conductance derived from (1) (and, *a fortiori*, all higher-order conductance moments) diverges for all sample lengths L, as first noted by Landauer [5]. This may be traced to the fact that the distribution of the dimensionless resistance defined by (1) is very broad and, for finite L, it has non-zero amplitudes at $\rho = 0$, corresponding to near transparency. This property of the distribution of ρ is also strikingly illustrated in a recent numerical study [6], for arbitrary L (see figure 4 of [6]). The divergence of the conductance moments led Anderson and co-workers [7] to suggest that the conductance that is observed in an experiment corresponds to an appropriate (finite) typical conductance†, rather than to the mean conductance.

Following the appearance of [1], [3], and [4] some detailed derivations of the Landauer formula from the linear response formalism (Kubo formula) have been presented [8, 9], together with strong criticisms of the formula $g = t$ [9, 10]. However, the contro-

† After identifying $\ln \rho$ as a self-averaging variable (with $\langle \ln \rho \rangle \sim L$) for $L \to \infty$, they define a typical conductance $g_s = \exp(-\langle \ln \rho \rangle)$.

versy concerning the definition of g may not be resolved inasmuch as this definition depends on what the disordered chain is connected to [3, 8, 9]. Thus, some authors [11, 12] continue to assume that g is proportional to t. While this has the drawback that the conductance of a perfect conductor is restricted to values of order unity, rather than being infinite, it has the advantage of leading to a mean conductance [1] having the exponentially decaying form of the typical conductance [7] for $L \to \infty$. We believe that further detailed studies of the statistical properties of t and their comparison with experimental results on conductance might help to resolve the above controversy from a practical point of view.

Detailed microscopic studies of the statistics of the transmission coefficient in one-dimensional systems described by a Gaussian δ-correlated random potential [1, 2] have been restricted to the low-transmission regime corresponding to $L \gg L_c$ (L_c = localisation length). The same is true for their recent generalisation to arbitrary disorder [12]. The purpose of the present Letter is to supplement the results of Abrikosov and Mel'nikov [1, 2] by analysing the distribution of t in the high-transmission (or quasi-metallic) regime corresponding to $L \ll L_c$, and to study the effect of a constant electric field on both high- and low-transmission statistics. As is well known [13], a strong electric field has the effect of spreading out the wavefunctions in one dimension from an exponentially localised form to a weaker power-law localised form, thus leading to important qualitative changes of the various moments of t. Our analysis is based on the method of invariant imbedding, which has been extensively used of late in the context of resistance fluctuations [14, 15].

In the framework of the imbedding procedure it may be shown [16] that the amplitude on an electronic wave reflected by a random conductor of length L, in the presence of a constant electric field $\mathscr{E} = -F$ of strength F, is given by

$$2\mathrm{i}k_1\,\mathrm{d}R/\mathrm{d}L = -k^2(1+R)^2 + k_1^2(1-R)^2 + \mathrm{i}(1-R^2)\,\mathrm{d}k_1/\mathrm{d}L \tag{2}$$

where

$$k_1 = (k_0^2 + 2|e|FL)^{1/2} \tag{3}$$

is the wavenumber of an incident electron in the region $x > L$ to the right of the conductor, k_0 is the wavenumber of a transmitted electron of energy $E = \frac{1}{2}k_0^2$ (in units with $\hbar = m = 1$) in the region $x < 0$ and, finally,

$$k^2 = k_0^2 - 2(V(L) - |e|FL) \tag{4}$$

where $V(L)$ is the random potential at the right-hand edge of the sample, which we assume to be Gaussian and δ-correlated:

$$\langle V(L)V(L')\rangle = V_0^2\delta(L-L') \qquad \langle V(L)\rangle = 0. \tag{5}$$

Equation (2) generalises the form of the imbedding equation for $R(L)$ [17] in the case where the incident wavenumber differs from the transmitted wavenumber by an L-dependent term, rather than by a constant.

We write the complex reflection amplitude in the form $R = (1-t)^{1/2}\exp \mathrm{i}\theta$ and transform equation (2) into separate equations for the transmission coefficient, $t = 1 - |R|^2$, and the phase θ:

$$\mathrm{d}t/\mathrm{d}L = [(2V(L)/k_1)\sin\theta - (|e|F/k_1^2)\cos\theta]\,t(1-t)^{1/2} \tag{6}$$

$$\mathrm{d}\theta/\mathrm{d}L = -[(V(L)/k_1)\cos\theta + (|e|F/2k_1^2)\sin\theta](2-t)/(1-t)^{1/2}$$

$$- (2/k_1)(V(L) - k_1^2). \tag{7}$$

Next we derive recursion relations for the moments of t, $t_n \equiv \langle t^n \rangle$, $n = 1, 2, \ldots$, by transforming (6) into an equation for the nth power of t and averaging over the random potential (5). Averages of quantities of the form $V(L)f(t, \theta)$, where f depends implicitly on $V(L)$ through t and θ, are performed by using Novikov's formula [18]

$$\langle V(L)f(t, \theta)\rangle = \tfrac{1}{2}V_0^2\langle[(\partial f/\partial t)\,\delta t/\delta V(L) + (\delta f/\partial\theta)\,\delta\theta/\delta V(L)]\rangle \tag{8}$$

where the variational derivatives of t and θ are readily obtained from the first integrals of equations (6) and (7). Finally, we assume, as usual, that the phase θ is an independent random variable, uniformly distributed [7] between 0 and 2π, which allows us to decouple averages: $\langle g(\theta)h(t)\rangle = \langle g(\theta)\rangle\langle h(t)\rangle$, and to evaluate $\langle g(\theta)\rangle$ explicitly. In this way we obtain the desired recursion relations

$$\mathrm{d}t_n/\mathrm{d}l = n(n-1)t_n - n^2 t_{n+1} \qquad n = 0, 1, 2, \ldots \tag{9}$$

where we have defined the dimensionless reduced length

$$l = V_0^2 \int_0^L \frac{\mathrm{d}L}{k_1^2} = \frac{V_0^2}{2|e|F} \ln\left(1 + \frac{2|e|FL}{k_0^2}\right) \tag{10}$$

whose expression for low fields, $2|e|FLk_0^{-2} \ll 1$, is

$$l = (L/L_c)(1 - |e|FL/k_0^2 + \ldots) \tag{10a}$$

where $L_c = k_0^2/V_0^2$ is the zero-field localisation length. The system (9) is not convenient for solution (which requires the explicit form of t_1!) and, as in previous work [1, 2], we shall derive the moments of t from the probability distribution of t^{-1} defined in terms of moments $u_n \equiv \langle t^{-n}\rangle$. The stochastic equation for the inverse transmission coefficient $u \equiv t^{-1}$ is readily obtained from (6), and by following the procedure outlined above, we get

$$\mathrm{d}u_n/\mathrm{d}l = n(n+1)u_n - n^2 u_{n-1} \qquad n = 0, 1, 2, \ldots . \tag{11}$$

Actually, it is also quite easy to derive a differential equation for the full distribution, $P_u(u, l)$, from the stochastic equation for u. Rewriting the latter for an arbitrary function $f(u)$

$$\mathrm{d}f/\mathrm{d}L = -[(2V(L)/k_1)\sin\theta - (|e|F/k_1^2)\cos\theta][u(u-1)]^{1/2}\,\mathrm{d}f/\mathrm{d}u \tag{12}$$

and averaging as discussed above, assuming uniformly distributed phases, we obtain

$$\mathrm{d}\langle f\rangle/\mathrm{d}l = \langle[(2u-1)\,\mathrm{d}f/\mathrm{d}u + u(u-1)\,\mathrm{d}^2 f/\mathrm{d}u^2]\rangle \tag{13}$$

where l is given by (10). Finally, using the definition of averages in terms of the distribution $P_u(u, l)$ of u, we obtain a Fokker–Planck-type equation for $P_u(u, l)$ by partial integration of the terms on the right hand side of (13):

$$\partial P_u/\partial l = \partial[u(u-1)\,\partial P_u/\partial u]/\partial u \tag{14}$$

which differs from equation (8) in [1] by the definition of l, which now includes the effect of the electric field. It is comforting to know that the method of invariant imbedding leads to the same equation as Abrikosov's quite different method for $F = 0$. The analogous equations for the distribution of the transmission coefficient and of the resistance defined by (1) may be readily obtained by the same procedure, starting from (6)†. We also emphasise that equations (9), (11) and (14) are exact under the assumption of uniformly distributed phases and, in particular, they are valid for arbitrary fields. Equation (1) may also be obtained from (14) by multiplying both sides by u^n and, after integrating over u, transforming the terms on the right hand side by partial integrations.

We restrict detailed discussion of the results for the transmission coefficient obtained from the above treatment to two important aspects which have previously not been analysed: the form of the field-dependent distribution of t in the quasi-metallic (or high-transmission) regime, and the effect of an electric field on the moments, particularly in the low-transmission regime where its delocalising effect on the electron wavefunctions is most visible.

The quasi-metallic regime, $l \ll 1$, owes its name to the relation $\langle \rho \rangle \sim l \to 0$, which implies diffusive motion of electrons at zero field [5]. By expanding the exact solutions of the system (11), $u_1 = \frac{1}{2}(e^{2l}+1)$, $u_2 = \frac{1}{6}e^{6l} + \frac{1}{2}e^{2l} + \frac{1}{3}$, $u_3 = \frac{1}{20}e^{12l} + \frac{1}{4}e^{6l} + \frac{9}{20}e^{2l} + \frac{1}{4}$, etc; for $l \to 0$ we get $u_n = 1 + nl + n^2l^2 + \ldots$, $n = 1, 2, \ldots$, which suggests that the asymptotic solution of (11) for $nl \ll 1$ is

$$\lim_{l\to 0} u_n = (1-nl)^{-1} \qquad n = 0, 1, 2, \ldots . \tag{15}$$

Substitution of this expression into (11) shows that it has the same degree of accuracy as its Taylor expansion up to cubic order. By using the identity

$$(1-nl)^{-1} = \int_0^\infty \mathrm{d}s \exp[-s(1-nl)]$$

to perform the summation over the moments in the characteristic function, we finally obtain the distribution

$$P_u(u,l) = l^{-1}u^{(-1/l-1)} \qquad l \to 0. \tag{16}$$

The distribution of the transmission coefficient is then

$$P_t(t,l) = \int_1^\infty \mathrm{d}u\, P_u(u,l)\delta(t-u^{-1}) = l^{-1}t^{(1/l-1)} \qquad l \to 0 \tag{17}$$

whose moments are $t_n = (1+nl)^{-1}$, $n = 0, 1, 2, \ldots$. The power-law form of the distributions (16) and (17) contrasts with the form of the distribution of resistance in the high-transmission regime. The latter is found by expanding the Landauer formula (1) for $t \to 1$,

$$\rho = t^{-1} - 1 = -\ln t + \mathrm{O}(\ln^2 t). \tag{18}$$

From (17) and (18) one then obtains

$$P_\rho(\rho, l) \simeq l^{-1}\exp(-\rho/l). \tag{19}$$

An expression of the same form for the distribution of resistance in the presence of an electric field has been obtained recently [6], using an involved but powerful transfer-matrix method. However, this approach is phenomenological in that it introduces a dimensionless length l proportional to L, whose explicit form can only be obtained from an intuitive argument [13], or by comparison with the results of a microscopic treatment

† The equation for the distribution of resistance ρ derived earlier [14], using Van Kampen's lemma, includes an additional field-dependent term on the right hand side. This term arises from their use of an imbedding equation which is invalid when the incident wavenumber k_1 depends explicitly on L [17].

such as the one given here. Finally, we note that the relative variance also has a characteristic form, of $(1/t_1)(t_2 - t_1^2)^{1/2} = l + O(l^2)$; it varies linearly with L at low fields and logarithmically at strong fields.

Next we consider the low-transmission regime that obtains for $l \gg 1$. This regime has been thoroughly studied at zero field in [1] and in [2] and, more recently, in [11] and [12]. Since the electric field enters only through the definition of l we may use the results of Abrikosov and we have

$$u_n \simeq [n!/2^n(2n-1)!] \exp[n(n+1)l] \tag{20}$$

$$t_n \simeq \tfrac{1}{2}\pi^{3/2}(\Gamma^2(n-\tfrac{1}{2})/\Gamma^2(n))\, l^{-3/2} \exp(-\tfrac{1}{4}l) \qquad n = 1, 2, \ldots \qquad l \gg 1. \tag{21}$$

We note that, while the analysis leading to (21) is relatively involved, the form of the leading exponential decrease (but not the correct pre-exponential coefficient) may readily be found by evaluating t_n using the asymptotic log–normal distribution for $u = t^{-1}$ (equation (19) of [1]) obtained from (14). At low fields u_n and t_n vary exponentially in the first approximation, while at strong fields one obtains power-law variations: u_n increases as

$$u_n \sim L^{\alpha_n(F)} \qquad \alpha_n(F) = n(n+1)V_0^2/2|e|F \tag{22}$$

and t_n decreases as

$$t_n \sim \ln^{-3/2}(2k_0^{-2}|e|FL)(2k_0^{-2}|e|FL)^{-V_0^2/8|e|F} \qquad l \gg 1 \tag{23}$$

which implies asymptotic power-law localisation. In particular, as in the zero-field case [1], the relative variances of u and t obtained from (22) and (23) are qualitatively the same and indicate that both quantities remain non-self-averaging when F is increased to large values.

The slowing down of the variations of u_n and t_n with increasing L, for $L \to \infty$, is related to the spreading of wavefunctions of the localised states under the influence of an electric field. We note that another way of increasing the conductance of a disordered electronic system is to use finite frequencies. This is particularly obvious in an infinite one-dimensional sample where the DC conductivity is zero (due to the localised nature of the eigenstates), while the AC conductivity is not. A recent study [19] of the L-dependence of the frequency-dependent conductance [4], $g(\omega, L)$, of a disordered one-dimensional conductor indeed shows a slowing-down of the decay of the mean conductance from an e^{-L} form at $\omega = 0$ to a form proportional to L^{-2} at $\omega \neq 0$. On the other hand, the mean of $\ln g(\omega, L)$ is slowed down from a linear form at $\omega = 0$ to a logarithmic form at $\omega \neq 0$. The latter behaviour is similar to the asymptotic variation of $\langle \ln t \rangle$ in a constant electric field. Indeed, from the Gaussian distribution of $\ln t = -\ln u$ in the interval $-\infty \leq \ln t \leq 0$ for $L \to \infty$ [1], one has

$$\langle \ln t \rangle = -l + O[l^{1/2} \exp(-l/4)] \tag{24a}$$

which decreases as $-L$ at low fields and as $-\ln L$ at strong fields (equation (10)). We note that (24*a*) coincides to leading order with an equation derived intuitively by Soukoulis and co-workers [13]. Similarly, the relative variance, given by

$$(\mathrm{Var} \ln t)/\langle \ln t \rangle^2 \sim -2/l \tag{24b}$$

crosses over from an L^{-1}-behaviour at low fields to an $\ln^{-1} L$-behaviour at strong fields, which may be compared with the corresponding form for the frequency-dependent conductance [19]:

$$(\mathrm{Var}\ln g(\omega, L))/\langle\ln g(\omega, L)\rangle^2 \sim \ln^{-2} L \qquad L\rightarrow\infty \qquad \omega\neq 0.$$

Finally, from the form of the absolute variance, Var ln $t \simeq 2l$, it follows that fluctuations are diminished in a weak electric field [14].

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